MVE055/MSG810, Matematisk statistik och diskret matematik, 2016/17
Assignment 1
Due September 19, 2016

1. Anna checks the weather report every day before deciding whether to carry an umbrella or not. If the forecast is "rain", the probability of actually having rain that day is 70%. If the forecast is "no rain", the probability of it actually raining is 20%. During fall and winter the forecast is rain 80% of the time and during summer and spring it is 10%.

(a) Assume that one day Anna missed the forecast and it rained. What is the probability that the forecast was "rain" if it was during the winter? What is the probability that the forecast was "rain" if it was during the summer?

(b) Assume that the probability of Anna missing the morning forecast is equal to 0.2 on any day in the year. If she misses the forecast, she will flip a fair coin to decide whether to carry an umbrella. On any day of a given season she sees the forecast, if it says "rain" she will always carry an umbrella, and if it says "no rain", she will not carry an umbrella. Are the events "Anna is carrying an umbrella", and "The forecast is no rain" independent? Does your answer depend on the season?

(c) Anna is carrying an umbrella and it is not raining. What is the probability that she saw the forecast? Does it depend on the season?

2. Assume that I_A is the indicator of an event A (indicator random variable), which is denoted by:

$$I_A(w) = \begin{cases} 1, & \text{if } w \in A, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Prove that two events A and B are independent if and only if the associated indicator random variables, I_A and I_B are independent.

(b) Show that if $X = I_A$, then $E[X] = P(A)$.

3. Assume that X_1 and X_2 are iid random variables and have Geometric distribution with success probability p. Show that for $i = 1, 2, ..., n-1$,

$$P(X_1 = i | X_1 + X_2 = n) = \frac{1}{n-1}$$